

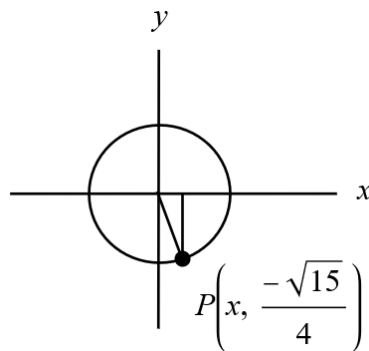
Exercise 138

For the following exercises, P is a point on the unit circle. a. Find the (exact) missing coordinate value of each point and b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P . Rationalize denominators.

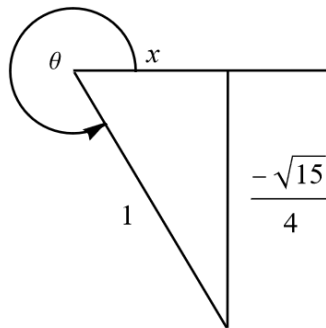
$$P\left(x, \frac{-\sqrt{15}}{4}\right), x > 0$$

Solution

The given point P on the unit circle is shown below. $x > 0$ means that it's in the right half.



Zoom in on the right triangle formed by P . θ is the counterclockwise angle from the positive x -axis.



The hypotenuse has a length of 1 because P is on the unit circle. The sides of a right triangle are related by the Pythagorean theorem, and this allows us to determine x .

$$x^2 + \left(\frac{-\sqrt{15}}{4}\right)^2 = 1^2$$

$$x^2 = 1^2 - \left(\frac{-\sqrt{15}}{4}\right)^2$$

$$x^2 = \frac{1}{16} \quad \rightarrow \quad x = \frac{1}{4}$$

Therefore, the six trigonometric functions are

$$\sin \theta = \frac{-\frac{\sqrt{15}}{4}}{1} = -\frac{\sqrt{15}}{4}$$

$$\cos \theta = \frac{x}{1} = x = \frac{1}{4}$$

$$\tan \theta = \frac{-\frac{\sqrt{15}}{4}}{x} = \frac{-\frac{\sqrt{15}}{4}}{\frac{1}{4}} = -\sqrt{15}$$

$$\csc \theta = \frac{1}{-\frac{\sqrt{15}}{4}} = -\frac{4}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\sec \theta = \frac{1}{x} = \frac{1}{\frac{1}{4}} = 4$$

$$\cot \theta = \frac{x}{-\frac{\sqrt{15}}{4}} = \frac{\frac{1}{4}}{-\frac{\sqrt{15}}{4}} = -\frac{1}{\sqrt{15}} = -\frac{\sqrt{15}}{15}.$$